Relative Controllability of Functional Differential Systems of Sobolev Type in Banach Spaces

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Abstract. In this paper, the relative controllability of functional differential systems of Sobolev Type in Banach Spacesof the form

$$(Ex(t))' + Ax(t) = Bu(t) + f(t, x_t), t > 0 \qquad (1.1)$$

was presented for controllability analysis. For purposes of clarity, we defined and extracted the following terminologies as they relate to system (1.1) from the solution given as integral formula [system (1.2)] **- complete set, reachable set, attainable set, target set , properness and relative controllability**. Necessary and sufficient conditions for the system (1.1) to be relatively controllable are established using Theorem 2.1 and Theorem 2.2. That is, (a) The system is relatively controllable if and only is zero is in the interior of the reachable set(b) The system is relatively controllable if the controllability grammian (map) of system (1.1) is non-singular.The results are established using the controllability standard-properness. The reachable set, attainable set and target set upon which our results hinge are extracted from the system (1.2) or the solution of system (1.1).

Keywords: Relatively Controllable, Relative Controllability, Functional Differential Systems, Sobolev Spaces, Banach Spaces. Reachable Set, Attainable Set and Target Set.

1.INTRODUCTION

The controllability of functional differential systems of Sobolev type in Banach Spaces had been established by KRISHNAN BALACHANDRAN AND JERALD DAUER(1998).

According toKrishnan Balachandran And Jerald Dauer(1998),the problem of controllability of linear and nonlinear systems represented by ordinary differential equations infinite dimensional spaces has been extensively studied. Several authorizes (see E.N. Chukwu (1991), R.F. curtain and A.J. Prichard (1978), I. Lasieka and R. Triggiani (1991) have extended the concept of infinite dimensional systems in Banach spaces with bounded operators. R. Triggiani (1975) established sufficient conditions for controllability of linear and nonlinear systems in Banach spaces with bounded operators. R. Triggiani (1975) established sufficient conditions for controllability of linear and nonlinear systems in Banach Spaces. Exact controllability of abstract semilinear equations has been Studied by I. Labiecka and R. Triggiani (1991). Y.C. Kwun, T.Y. Park and J.W. Ryu (1991) investigated the controllability and approximate controllability of delay volterra systems by using fixed point theorem. K. Balachandran, P. Balasubramaniam and J.P. Daner (1995), K. Balachandran, P. Balasubramaniam and J.P. Dauer (1995) and K. Balachandran, P. Balasubramaniam and J.P. Dauer (1996) studied the controllability and local null controllability of nonlinear integro-differential systems and functional differential systems in Banach Spaces and it was shown that controllability problem in Banach Spaces can be converted into one of a fixed-point problem for a single le-valued mapping. It is known from Onwuatu, J.U. (1993) that if a system is relatively controllable, the optimal control is unique and Ban-Bang. In the light of this, we shall consider the functional differential systems of Sobolev Type in Banach Spaces of the form.

$$(E\,x\,(t))' + A\,x\,(t) = Bu(t) + f(t,x_t), t > 0 \qquad (1.1)$$

(a nonlinear partial functional differential system) where the state x(.) takes values in a Banach Spaces X and the control function u(.) is given in $L^2(J, U)$, the Banach Spaces of admissible control functions with U a Banach Space. B is a bounded linear operator from U into Y, a Banach Space. The nonlinear operator F: JxC → Y is continuous. Here J = [0, t_1] and for a continuous function $x: J* = [-l, t_1] \rightarrow X, x_t$ *is that element of* $C = C([-l,0]:X)$ *defined by* $x_t(s) = x(t+s); -\iota \le s \le 0.$

The domain of E D(E) becomes a Banach Space with norm $\|x\|_{D(E)} = \|Ex\|_Y, x \epsilon D(E)$ *and* $C(E) = C([-\iota, 0]:D(E)).$

The above system (1.1) will be investigated for **relative controllability, existence, form and uniqueness of optimal control** by first of all considering the relative controllability of the system. For a given admissible control u(t) there exists a unique solution $x(t,u)$ *for* $t \epsilon (0, t_1)$ of the system (1.1) described by the integral formula , see Krishnan Balachandran and Jerald P. Dauer (1998).

$$x(t) = E^{-1}T(t)E\Phi(0) + \int_0^t E^{-1}T(t-s)f(s,x_s)ds + \int_0^t E^{-1}T(t-s)Bu(s)ds$$
$$x(t) = \Phi(t), -\iota \le t \le 0. \qquad (1.2)$$

For purposes of clarity, we define the following terminologies as they relate to system (1.1). With the solution given as integral formula [system (1.2)], we define **complete set, reachable set, attainable set, target set , properness and relative controllability**.

Definition 1.1 (complete state)

The complete state for system (1.1) is given by the *set*

$$z(t) = \{x, u_t\}$$

Definition 1.2 (Reachable Set)

The reachable for the system (1.1) is given as

$$R(t_1, 0) = \left\{\int_0^t E^{-1}T(t,s)\,Bu(s)\,ds\right\}$$

Definition 1.3 (Attainable Set)

Attainable set is the set of all possible solutions of a given control system. In the case of the system (1.1), for instance, it is given as

$$A\,(t_1, 0),$$
$$= \left\{x\,(t) = E^{-1}T\,(t)\,E\,\Phi(0) + \int_0^t E^{-1}T\,(t-s)\,f(s,x_s)ds + \int_0^t E^{-1}T\,(t-s\,B(s)u(s)ds;\ u\epsilon U\right\}$$

$$= E^{-1}\left\{T\,(t)[E\Phi(0)] + \int_0^t T\,(t-s)\,f(s,x_s)ds + \int_0^t T\,(t-s\,B(s)u(s)ds\ ;u\epsilon U\right\}$$
$$= \{\eta + R\,(t_1, 0)\}.$$
$$where\ \eta = E^{-1}T\,(t)\,E\,\Phi(0) + \int_0^t E^{-1}T\,(t-s)f(s,x_s)ds$$

Since E (t) is a fundamental matrix and fundamental matrices are invertible, E^{-1} *exists*.

Or

Attainable Set for the system (1.1) is given as

$\mathbf{A}\ (t_1, 0) = \{x(t,u): u\epsilon U\}, where\ U = \{u\epsilon L_2\ ([0,t_1], X): |u_j| \leq\ 1\ ;\ j = 1, 2, \ldots, m\}$.

Definition 1.4(Target Set)

The target set for system (1.1) *denoted by* $G\ (t_1, 0)$ *is given as*

$G\ (t_1, 0) = \{x(t) = x\ (t,u): t_1\ \geq \tau > 0\ for\ fixed\ \tau\ and\ u\epsilon U\}$.

Definition 1.5 (Properness)

The system (1.1) *is proper in* X *on* $(0, t_1)$ *if and only if.*

$C^T [E^{-1} T\ (t - s) B(s)] = 0\ (almost\ every\ where)\ t_1 > 0 \Rightarrow C = 0, C\epsilon X.$

Definition 1.6 (Controllability Grammian)

The controllability grammian for the system (1.1) is given as

$$W\ (t_1, 0) = \int_0^t z(t,s)\, z^T (t,s) ds$$

Where $z(t,s) = [E^{-1} T(t - s) B(s)]$ *and* T *denotes matrix transpase .*

Definition 1.7 (Relative Controllability)

The system (1.1)*is said to be relatively controllable on* $[0, t_1]$ *if for every initial complete state* $z(0)$ *and* $x_1 \epsilon X$, *there exists a control function* $u(t)$ *defined in* $[0, t_1]$ *such that the solution* (1.2) *of the system* (1.1) *satisfies* $x(t_1) = x_1$.

2. Main Results

We now state and prove the following theorems that guarantee relative controllability of the system (1.1) under study.

Theorem 2.1.(Necessary Condition).

consider the system (1.1) given as

$$(E\, x(t))^{1} + A\, x(t) = Bu(t) + f(t, x_t), t > 0 \tag{2.1}$$

With the same conditions on the system parameters as in (1.1), then the following statements are equivalent:

(1) ***System*** $(\mathbf{1.1})$ ***is relatively controllable on*** $[0, t_1]$

(2) ***The controllability grammian*** $W(t, 0)$ ***of system*** $(\mathbf{1.1})$ ***is nonsingular.***

(3) ***The system*** $(\mathbf{1.1})$ ***is proper on*** $[0, t_1]$.

Proof

Straight forward from the arguments in, Anajevskii and Kolmanovskii (1990), Angell (1990), Balachandran and Dauer (2002).

Theorem 2.2 (Sufficient Condition)

The system (1.1) with its standing hypothesis is relatively controllable if and only if zero is in the interior of the reachable set. That is system (1.1) is relatively controllable if and only if.

$$0 \epsilon Int\ R\ (t_1, 0) for\ t_1 > 0.$$

Proof

The reachable set $R(t_1, 0)$ *is a closed ,convex and compact subset of* X. Therefore, a *point* $z_1 \epsilon X$ *on the boundary implies there is a support plane* π *of* $R(t_1, 0)$ *through* z_1.

That is,

$C^T(z - z_1) < 0$ *for each* $z \epsilon R(t_1, 0)$,
where $C \neq 0$ *is an outward normal to the support plane* π.
If u_1 *is the corresponding control to* z_1, *we have*
$C^T \int_0^{t_1} [E^{-1}T(t-s)B(s)]u(s)ds \leq C^T \int_0^{t_1} [E^{-1}T(t-s)B(s)]u_1(s)ds$, *for each* $u \epsilon U$
(2.2)
Since U is a sphere, the inequality (2.2) becomes

$$\left|C^T \int_0^{t_1} [E^{-1}T(t-s)B(s)]u(s)ds\right| \leq \left|C^T \int_0^{t_1} [E^{-1}T(t-s)B(s)].1\right| ds$$

$$= C^T \int_0^{t_1} [E^{-1}T(t-s)B(s).1] \mathbf{sgn} C^T[E^{-1}T(t-s)B(s)]ds \qquad (2.3)$$

Compare equation (2.2) with equation (2.3) we have

$$u_1(t) = sgn C^T[E^{-1}T(t-s)B(s)]$$

More so, as z_1 *is on the boundary, since we always have* $0 \epsilon R(t_1, 0)$.
If 0 *were not in the interior of* $R(t_1, 0)$, *then it is on the boundary*.
Hence, from the preceding argument it implies that

$$0 = C^T \int_0^{t_1} [E^{-1}T(t-s)B(s)]\, ds$$

So that
$C^T[E^{-1}T(t-s)B(s)] = 0$, almost everywhere.
Then, by the definition of properness, this implies that the system is not proper, *since* $C^T \neq 0$.
However, if 0ϵ *Interior* $R(t_1, 0)$ *for* $t_1 > 0$

$$C^T[E^{-1}T(t-s)B(s)] = 0, \Rightarrow C = 0,$$

which is the properness of the system (1.1) and by the equivalence in theorem 2.1, the relative controllability of the system (1.1) on the interval $[0, t_1]$ *is proved* .

CONCLUSION

We have established Necessary and Sufficient Conditions for Functional Differential Systems of Sobolev Type in Banach Spaces to be Relatively Controllable, using the Controllability Standards-(Properness of the Systems, and Zero being in the interior of the Reachable Set) and theorem 2.1 / theorem 2.2.

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