Cross Correlation Function for Different Networks Dimension

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Abstract- Determination of Dimensionality of Wireless Sensor Network (WSN) is very significant for proper setup of the network. A procedure using Cross Correlation Function (CCF) can be used to find it. A network with several sensors is considered where sensors are arranged in different shapes representing different dimensions. Then CCF of these networks is founded out. From the output of the CCF to these networks, a decision can be made about the dimensionality. After that, using analytical analysis this procedure is compared with the simulation result. There exists a great similarity between the simulation and analytical results which give a conclusion in the favour of using the procedure in estimating the dimensionality of WSN.

Keywords- Network dimensionality, Wireless Sensor Network (WSN), Cross Correlation Function (CCF), Dirac delta function.

1 INTRODUCTION

Currently, with the increased advancement in the technology WSN is widely used for national security, finding out natural resources, weather forecasting, communication in remote area, etc. This WSN can also be applied to the underwater for collecting information about disaster alleviation (Soreide, Woody, & Holt, 2001), pollution detection (M. Anower, Chowdhury, Giti, Sayem, & Haque, 2014), an underwater data collection (Chowdhury, Anower, & Giti, 2014), extracting information for scientific analysis (Chowdhury et al., 2014). These prospects will find a success with proper operation of the network, which has a great dependency on the dimensionality of the network.
Dimensionality of a WSN mainly defines the placement of the sensors which is called as nodes in this paper. The placements of the nodes determine proper operation of that network. A deployment strategy for 2D and 3D underwater acoustic network is proposed in (Pompili, Melodia, & Akyildiz, 2009) to determine the minimum number of sensors to be deployed to achieve optimal sensing and communication coverage. In (M. S. Anower, 2011) , a process to determine active nodes of Underwater Wireless Sensor Network (UWSN) using CCF is proposed. As the node placement defines the dimensionality so by using the CCF, dimensionality can also be determined.
The output of the CCF depends on the dimensionality so with the variation of the dimensionality, the output of the CCF will also vary. Notifying this variation in the dirac deltas (output of the CCF) the dimensionality can be determined easily. In this work different dimension (1D, 2D, 3D) is considered. Using the variation in the dirac delta's formation as a parameter a decision can be made over the dimensionality of the network.

2 CROSS CORRELATION OF RANDOM SIGNALS

Determination of CCF is done by sending a probing request from probing nodes or receivers to the sensors or transmitting nodes at first, then a Gaussian signal is sent back to the receivers from each node. The response received at two receivers contains a delay difference. Then all signals from all the transmitting nodes are summed at receivers and cross correlation of the summed signal is found.

Let two sensors with location at x_1 and x_2 for 1D, (x_1, y_1) and (x_2, y_2) for 2D, (x_1, y_1, z_1) and (x_2, y_2, z_2) for 3D is placed. Probe request from these sensors is given to the 10000 transmitting nodes and they give back Gaussian signal as their response. Let the Gaussian signal, $s(t)$ is the response of n^{th} transmitter, which will reach to the two sensors with different time delay τ_1 and τ_2, then the signals at two sensors are:

$$s_{rn1}(t) = \alpha_{n1} s_n(t - \tau_{n1}) \tag{1}$$

$$s_{rn2}(t) = \alpha_{n2} s_n(t - \tau_{n2}) \tag{2}$$

Total signals received by two sensors are:

$$s_{r1}(t) = \sum_{n=1}^{N} \alpha_{n1} s_n(t - \tau_{n1}) \tag{3}$$

$$s_{r2}(t) = \sum_{n=1}^{N} \alpha_{n2} s_n(t - \tau_{n2}) \tag{4}$$

Where N=no. of transmitting nodes =10000.

The Cross Correlation of these signals is

$$C(T) = \int_{-\infty}^{+\infty} s_{r1}(t) s_{r2}(t - \tau) dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=1}^{N} \alpha_{n1} s_n(t - \tau_{n1}) \sum_{n=1}^{N} \alpha_{n2} s_n(t - \tau_{n2} - \tau)\ dt \tag{5}$$

The output of the CCF takes a form of a series of deltas. The possible positions of those deltas define by bins, b:

$$b = \frac{2 \times d_{DBS} \times S_R}{S_p} - 1 \tag{6}$$

From equation (6) it is clear that bins can vary by varying distance between sensors, d_{DBS} and sampling rate, S_R for a fixed medium for which velocity of propagation, S_p is fixed. In this case there are 11 bins that mean the possible positions of delta functions are 11.

3 CCF BY ANALYTICAL ANALYSIS

The cross correlation function of time delayed version of infinity long unity strength Gaussian signal can be expressed by a delta function, whose amplitude depends on the attenuations and position will be the delay difference of the signals from the center of the CCF.

Then the CCF for 1st node is:

$$C_1(\tau) = \alpha_{11} \alpha_{12} \delta(\tau - \left[\frac{d_{11} - d_{12}}{S_p}\right]) \tag{7}$$

Assuming the strength of the node signal is high enough to overcome the attenuations, so neglecting the attenuations CCF for 1^{st} node become:

$$C_1(\tau) = \delta(\tau - \left[\frac{d_{11} - d_{12}}{S_p}\right]) \tag{8}$$

Where d_{11}=Distance between 1^{st} node and 1^{st} receiver

d_{12}=Distance between 1^{st} node and 2^{nd} receiver

Similarly CCF for N^{th} node is:

$$C_N(\tau) = \delta(\tau - \left[\frac{d_{N1} - d_{N2}}{S_p}\right]) \tag{9}$$

Then the CCF for N number of nodes

$$C(\tau) = \sum_{n=1}^{N} \delta(\tau - \left[\frac{d_{n1} - d_{n2}}{S_p}\right]) \tag{10}$$

If the node number is greater than the bin, then the bins can be occupied by more than one delta, increases the amplitude of the deltas in the bins. So the CCF can be expressed in terms of bins as:

$$C(\tau) = \sum_{i=1}^{b} p_i \delta_i \quad (11)$$

Where, p_i=amplitude of the delta,δ_i in the i^{th}bin.

The above analytical analysis is verified by simulation in the following Figure 1, where N=32 and bin=19 is considered. As the number of node is larger than bin, so there is possibility that some bins can be occupied by more than one node and some bins can be empty for the time delay difference. From figure 1 p_i values are:

$p_1 = p_{19} = 4, p_4 = p_{10} = p_{13} = 3$, and so on.

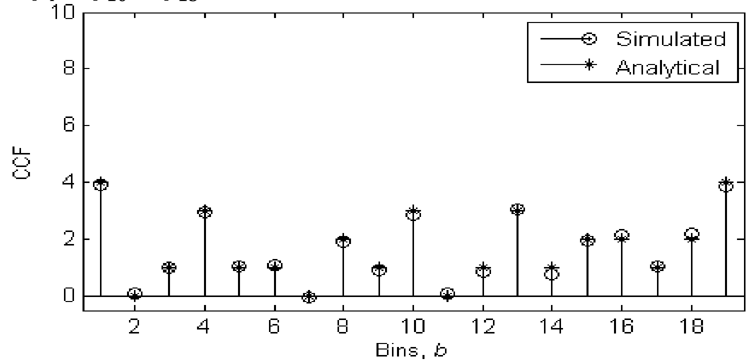


Figure 1: CCF for *N*=32, and *b*=19

Using moving average technique of cross correlation (Hanson & Yang, 2008a, 2008b), the CCF can be expressed generally by the following expression:

$$C(\tau) = \frac{1}{N_s-\tau}\sum_{i=1}^{N_s-\tau} x_i y_{i+\tau} - (\frac{1}{N_s}\sum_{i=1}^{N_s} x_i)(\frac{1}{N_s}\sum_{i=1}^{N_s} y_i) \quad (12)$$

Where N_s= Signal length in terms of samples

τ= Time delay of the Cross Correlated signals.

$x_i$andy_i are i^{th}samples of the two sensors' signals.

Assume Gaussian signal contain zero mean so the product of their mean is zero. So the CCF:

$$C(\tau) = \frac{1}{N_s-\tau}\sum_{i=1}^{N_s-\tau} x_i y_{i+\tau} \quad (13)$$

Using equation (13) the peak of deltas in b^{th} bin is given below:

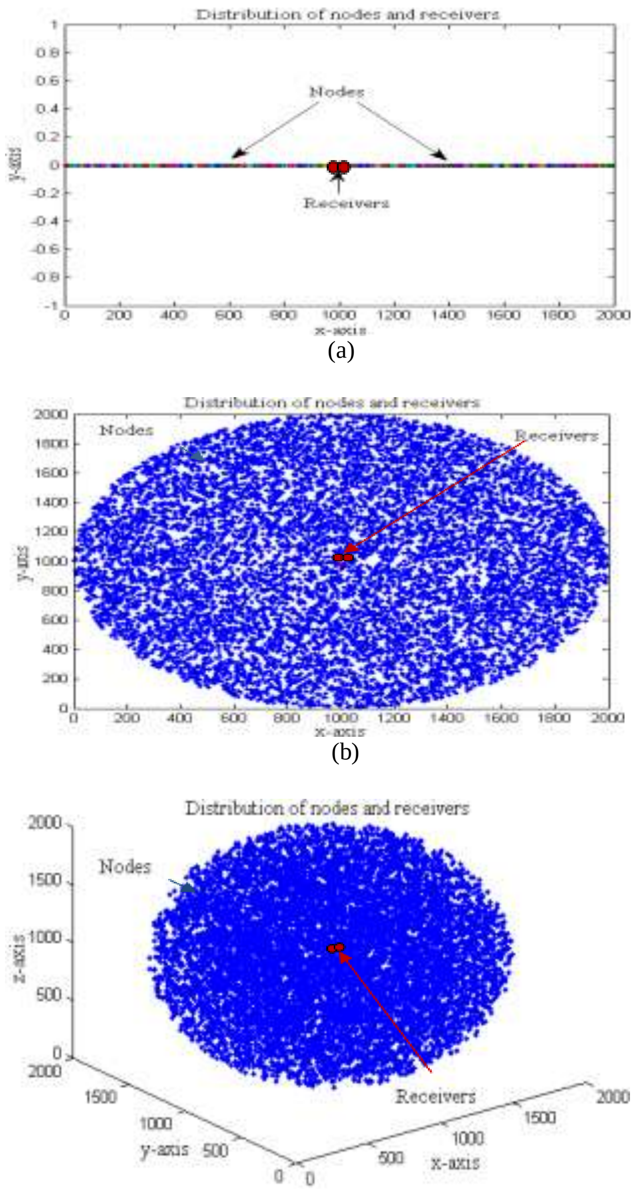
$$P_b = \frac{1}{N_s-\tau}\sum_{i=1}^{N_s-\tau} x_i y_{i+\tau} \quad (14)$$

Theoretical CCF is developed by putting these values to the equation (11).

4 CCF FOR DIFFERENT DIMENSIONS

4.1 Network Formation

In this work, three networks are designed with 10,000 transmitting nodes, which are distributed through a linear line for 1D network, along a circle and along a sphere for 2D network and 3D network respectively and two probing nodes or two receivers are placed at the centred position in each network. These receivers will measure the signal feedback coming from the transmitters. In the Figure 2 it is clearly shown that the uniform node distribution is a straight line, a circle and a sphere for 1D, 2D and 3D network respectively.



(a)

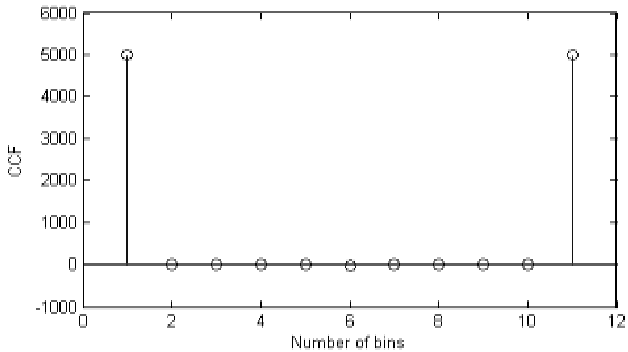
(b)

(c)

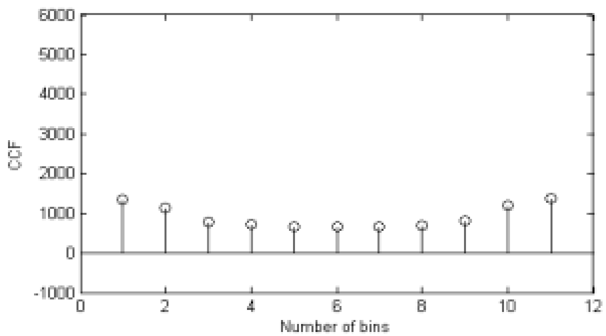
Figure2: Distributions of (10,000) nodes in (a) 1D; (b) 2D; and (c) 3D

4.2 CCF by Simulation

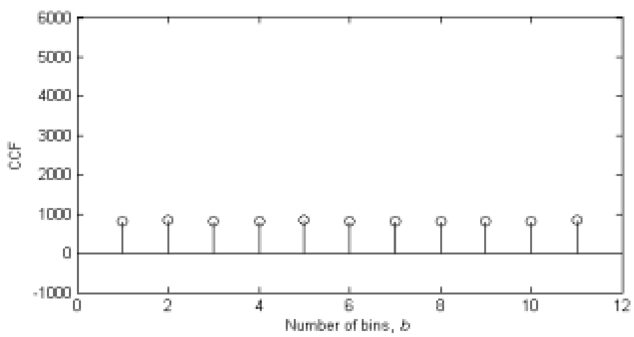
The constructed CCF from the simulation shows different results for different dimensions which is shown in Figure 3. CCF for 1D network contains only two dirac deltas at 1st and 11th bin with equal strength shown in Figure 3(a),CCF for 2D network is a series of dirac which contains a variation in the magnitude with a width of bin having centered at 6th bin of 11 bins shown in Figure 3(b),CCF for 3D network is a series of dirac deltas of uniform magnitude through the width $2d_{DBS}$ having centered at 0 shown in Figure 3(c).



(a)



(b)



(c)

Figure3: CCFs versus *b*: (a) 1D; (b) 2D; and (c) 3D

5 RESULTS AND DISCUSSION

Assuming the deltas and the no. of nodes are of equal strength, the analytical analysis using the equation (11) is plotted in Figure 4, with imposing the simulated result on it to find the comparison between the simulated and theoretical result. From Figure 4, it can be realized about the suitability about the estimation technique of the dimensionality using CCF.

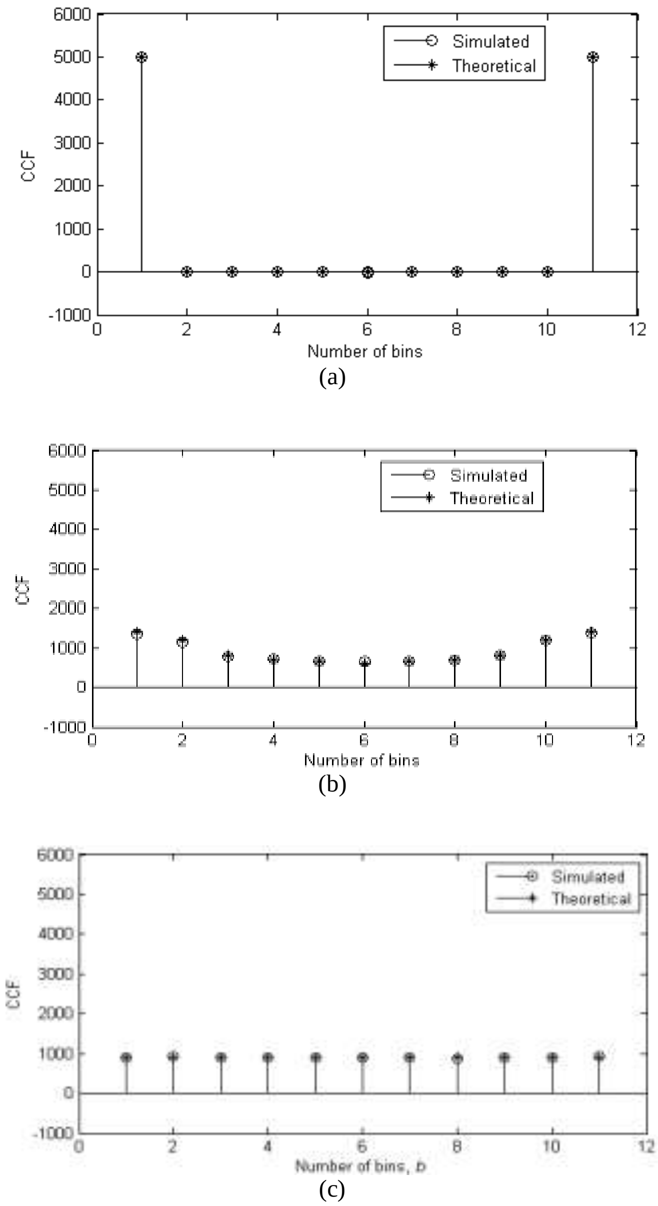


Figure 4: Simulated and theoretical distributions of CCFs:

(a) 1D; (b) 2D; and (c) 3D

6 CONCLUSION

In this paper a large number of transmitting nodes are used so it is easy to differentiate between the networks of 2D and 3D.As it is shown in the Figure 1 for a small number of nodes there will not be a uniform distribution of the deltas. In the absence of uniform distribution it will be difficult to differentiate between 2D and 3D network. Though it shows same CCF for 1D network, but for differentiating the CCF output results for 2D and 3D, it is must to have a large number of transmitting nodes. This is the major drawback of the process for finding the dimensionality of a network using Cross Correlation Function. In the future, we intend to overcome this limitation and determine the dimensionality for all types of network.

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