A fuzzy numbers approach for analyzing OECD well- being

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Abstract. The desire of human beings and the goal of government policy basically have a common point which is claimed as the well-being. However, the criteria heterogeneity between objective and subjective criteria causes difficulty in decision making. A fuzzy addition is thus proposed to give a vision on the significant information for the well-being. Empirically, the proposed approach applies dominance-based rough set approach on the well-being of Organization for Economic Co-operation and Development (OECD well- being) to disclose that the significant criteria for the top-ten nations are employment rate and life satisfaction

Keywords: *well- being, multiple criteria decision making (MCDM), fuzzy numbers, Organization for Economic Co-operation and Development (OECD).*

1 INTRODUCTION

A common point between the desire of human beings and the goal of government policy is claimed as the well-being of Organization for Economic Co-operation and Development (OECD-WB) which aims to raise the living standard (OECD, 2011, OECD, 2011 and N. Park and C. Peterson, 2009). . However, there are two potential problems have not been deeply explored. First, different-scaling criteria impose difficulties in understanding for stakeholders. Second, the aggregated and decomposed information is unknown or uncertain thus making the government policy hard to decide.

With the aforementioned problems, key challenges in analyzing the OECD-WB are summarized as the followings:

- OECD is the most well-known organization about the economic development. Recently, it provides the evaluation information comprised of 11 indexes and 24 criteria on the well-being. However, it does not provide significant criteria when considering objective and subjective living. In order to find the significant criteria, the induction technique can be helpful in providing conditional dependences for a decision.
- Dominance-based rough set approach (DRSA) can give induction features from an information system comprised of objective and subjective criteria (S. Greco, 2001, S. Greco, 2002 and R. Slowinski, 2009). However, it has a challenge, the induction quality decreases when the conjunctives increase, i.e., the feasible space very possible gets smaller.

To overcome the above challenges, this research proposes a fuzzy numbers approach illustrated in Fig. 1. It transforms objects within approximations into fuzzy numbers which associate criteria values with decision. These fuzzy numbers present a homogeneous-scaling to substitute different-scaling. Furthermore, the fuzzy numbers are added crossing criteria for an object. The addition operation not only calculates the dominance performance for objects but also leads the derivation of approximation boundaries which make classification available.

The fuzzy numbers of the approximation objects can preserve dominance characteristics. Therefore, users can easily get insight of dominating well-being. The methodology of this research has three stages. First, DRSA is applied to assign approximations and the fuzzy numbers. Second, the fuzzy number addition is used to provide membership degrees close to the decision. Third, the added fuzzy numbers are used to generate induction rules and provide features information.

This paper has the two main parts. The first is the implementation of the proposed methodology. The second is a study to get insight of the OECD-WB. The remainder of this paper is organized as follows: Section 2 reviews the well-being, DRSA, and the fuzzy numbers, Section 3 presents the propositions for the fuzzy numbers approach, Section 4 addresses results of the application, Section 5 presents discussions on FNA and the case study, and finally concluding remarks are presented to close the paper.

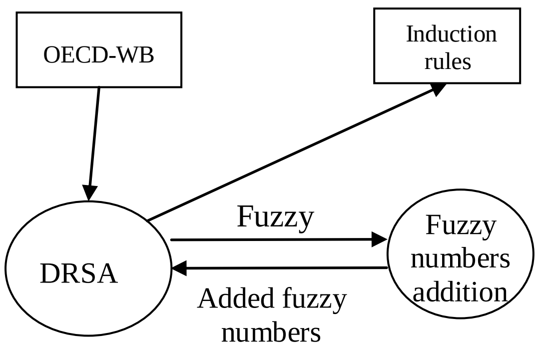


Fig. 1: The concept of fuzzy numbers approach

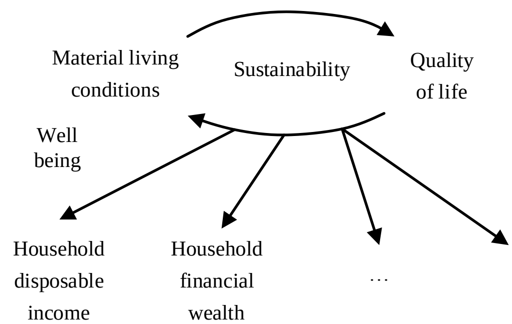


Fig. 2. The OECD well-being

2 LITERATURE REVIEW

The concept of well-being stems from the life meaning of Plato and the happiness of Aristotle (T. Metz, 2012 and R. Kraut, 2012) to social level and further national level. The ancient period emphasizes that the moral goods lead the value of the life meaning and the fulfillments of the moral goods lead people to happiness. Now the well-being becomes the focus of scholars and government (R. A. Easterlin, 1974, R. A. Easterlin, 1995 and L. A. King, 1998). Currently, OECD-WB is used to measure the living situation and government functioning. It has become the focus of policy making. For instance, Taiwan and Japan start to take the life satisfaction as benchmarking for government performance (www.esri.go.jp, 2012, news.chinatimes.com, 2012). Its structure is presented in Fig. 2. The criteria such as households' income, household financial wealth, etc., are proposed in three categories, i.e., quality of life, material living conditions, and sustainability over time. Totally, twenty four criteria are placed into eleven indexes as Table 1(OECD, 2012).

Table 1. Eleven indexes and twenty four criteria of OECD-WB in 2012

	Income		Community
q_1	Households' income	q_{17}	Social network
q_2	Household financial wealth		
	Jobs		**Civic engagement**
q_3	Employment rate	q_{18}	Consultation on rule-making
q_4	Personal earnings	q_{19}	Voter turn-out

q_5	Job Security		
q_6	Long-term unemployment rate		
	Housing		Environment
q_7	Rooms per person	q_{20}	Water quality
q_8	Housing expenditure	q_{21}	Air pollution
q_9	Dwellings with basic facilities		
	Work-life balance		Safety
q_{10}	Employees working very long hours	q_{22}	Homicide rate
q_{11}	Time devoted to leisure and personal care	q_{23}	Assault rate
	Health		Life Satisfaction
q_{12}	Life expectancy	q_{24}	Life Satisfaction
q_{13}	Self-reported health		
	Education		
q_{14}	Educational attainment		
q_{15}	Years in education		
q_{16}	Students' skills		

Facing OECD-WB the induction techniques is helpful to get inside knowledge. Therefore, the related techniques are reviewed in the followings.

2.1 DRSA:

DRSA is a powerful technique of relational structure and has been successfully applied in many fields [9-14]. In classification application, it can be used to induce objects assigned to $Cl_t^{\geq}$ (the upper ward union classes which include objects ranked at least t^{th}) or to $Cl_t^{<}$ (the downward union of classes which include objects ranked less than t^{th}), where Cl is a cluster set containing ordered classes Cl_t, $t \in T$ and $T = \{1,2,...,n\}$. The formulations for the above statement can be expressed as $Cl = \{Cl_1,...,Cl_t,...,Cl_n\}$, $Cl_1 = \{y \in U : y$ is ranked in the top position$\}$, $Cl_2 = \{y \in U : y$ is ranked in the second position$\}$, …, and $Cl_n = \{y \in U : y$ is ranked in the bottom position$\}$ where U is a set with decision makers' preference orders. For all $s,t \in T$ and $s \geq t$ (rank of s $\geq$ rank of t), every object in Cl_s is preferred to be at least as good as any of object in Cl_t. They are constructed as:

The dominating union: $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$ for $s \geq t$ and $Cl_t^{<} = \bigcup_{s<t} Cl_s$ for $s < t$

Another representation of the dominating set relies on a set of criteria, *P*. It follows the dominance principle of requiring each chosen object at least as good as a boundary object x in all considered criteria. The granules of a dominating set based on *P* can be viewed as the granular cones in the criteria value space. Vice versa the dominated sets follow the dominance principle and have granules in the opposite direction. These cones are categorized into *P*-dominating and *P*-dominated sets [26], respectively. It is said that object y P-dominates object x with respect to a criteria set P (denotation yD_Px).

Given $x, y \in U$ and P, let dominance sets as:

P- dominating set: $D_P^+(x) = \{y \in U, yD_Px\}$

P- dominated set: $D_P^-(x) = \{y \in U, xD_Py\}$

where $x, y \in Cl$, x plays a role for the boundary of $D_P^+(x)$ or $D_P^-(x)$, $y \succsim_q x$ for $D_P^+(x)$, $x \succsim_q y$ for $D_P^-(x)$, and all $q \in P$.

Two approximations are defined for illustrating the dominance consistency. The association between $Cl_t^{\geq}$ and P- dominating set should keep dominance consistency requiring $y \in Cl_t^{\geq}$ and $y \in P$- dominating.

$$\underline{P}(Cl_t^{\geq}) = \{x \in Cl_t^{\geq}, D_P^+(x) \subseteq Cl_t^{\geq}\} \qquad \overline{P}(Cl_t^{\geq}) = \bigcup_{x \in Cl_t^{\geq}} D_P^+(x),\ \boldsymbol{B}np(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$$

$$\underline{P}(Cl_t^{<}) = \{x \in Cl_t^{<}, D_P^-(x) \subseteq Cl_t^{<}\}\ \overline{P}(Cl_t^{<}) = \bigcup_{x \in Cl_t^{<}} D_P^-(x),\ \boldsymbol{B}np(Cl_t^{<}) = \overline{P}(Cl_t^{<}) - \underline{P}(Cl_t^{<})$$

where $t = 1, \ldots, n$, $\boldsymbol{B}np(Cl_t^{\geq})$ and $\boldsymbol{B}np(Cl_t^{<})$ are P-doubtful regions. $\underline{P}(Cl_t^{\geq})$ is defined by requiring that the largest union of P- dominating sets should be included in $Cl_t^{\geq}$. $\overline{P}(Cl_t^{\geq})$ is defined by requiring that the smallest union of P- dominating sets should contain all elements of $Cl_t^{\geq}$. These two approximations present the proper and possible assignments of objects into $Cl_t^{\geq}$ respectively. The objects belonging to the possible but not proper assignment are categorized as doubtful.

The proper assignments can be explained with the coverage rate defined by Pawlak (Z. Pawlak, 1997, Z. Pawlak, 2002) and Greco et al. [4, 5]. There are two typical coverage rates (CR) for the upward union $Cl_t^{\geq}$ and the downward union $Cl_t^{<}$, which are formulated as follows:

$$CR(Cl_t^{\geq}) = \frac{|\ \underline{P}(Cl_t^{\geq})\ |}{|\ Cl_t^{\geq}\ |} \text{ and } CR(Cl_t^{<}) = \frac{|\ \underline{P}(Cl_t^{<})\ |}{|\ Cl_t^{<}\ |}$$

The symbol CR is used to express "the probability of objects in the P-lower approximation relatively belonging to the corresponding union of decision classes." The possible assignment can be explained by the accuracy rate. Two typical accuracy rates (α) are listed as:

$$\alpha(Cl_t^{\geq}) = \frac{|\ \underline{P}(Cl_t^{\geq})\ |}{|\ \overline{P}(Cl_t^{\geq})\ |} = \frac{|\ \underline{P}(Cl_t^{\geq})\ |}{|\ U\ | - |\ \underline{P}(Cl_{t-1}^{<})\ |} \qquad \alpha(Cl_t^{<}) = \frac{|\ \underline{P}(Cl_t^{<})\ |}{|\ \overline{P}(Cl_t^{<})\ |} = \frac{|\ \underline{P}(Cl_t^{<})\ |}{|\ U\ | - |\ \underline{P}(Cl_{t+1}^{\geq})\ |}$$

The symbol α is used to present "a ratio of the cardinalities of P-lower approximation to those of P-upper approximation, i.e., the degree of the properly classified approximation relative to the possibly classified approximation." The relative importance of criteria in mathematics is reviewed next.

Saaty (2001) proposed that pair-wise comparisons and inductions can be formulated as ratios, and then transformed into the priority of criteria, or the criteria weights (T. L. Saaty, 2001). He also mentioned that the ratios represent how much approximately a criterion is as compared to another, and that its application can determine how close the criteria are. Also, he emphasized that ratio operations are independent from irrelevant alternatives. Thus the ratio scales derived from different (criteria) scales can be implemented mathematically to generate a characteristic ratio with invariance. Based on these theories, a multiplication of two ratios, the coverage and the accuracy rates, can be used to express an accuracy of the conditional probability.

2.2 Fuzzy numbers

A fuzzy number means an interval clustering around an objective. The membership function can assign each interval a monotonic value such that users can realize the degree of an object close to a decision (G. J. Klir nad B. Yuan, 1995, T. Terano, K. Asai, 1992 and G. J. Klir, U. St, 1997). The arithmetic operations on these intervals can express membership degree in wide perspectives. One operation related to this research is the addition operation. The related technique of the fuzzy numbers is described below.

Property 1: A fuzzy number addition:

$A+B=[a_1,a_2]+[b_1,b_2]=[a_1+b_1,a_2+b_2]$ where $A=[a_1,a_2]$, $B=[b_1,b_2]$, a_1,a_2,b_1,b_2 are interval boundaries, and A, B are the fuzzy numbers. The fuzzy addition can use a value to represent an aggregated concept crossing multiple criteria. The followings are fundamental properties related to the fuzzy numbers assignments by DRSA.

Property 2: Fuzzy cuts:

${}^{\alpha}A=\{x\in U\mid A(x)\geq\alpha,\ \alpha\in R,\ \alpha\geq 0\}$. $\alpha-cut$ makes dominating objects available to users. This can lead understanding of dominating group which is discussed in Section 3.

3 THE FUZZY NUMBERS ADDITION

The default ranks of the OECD-WB can be obtained through equal weights of criteria. This research takes the default ranks from the web site of OECD to identify significant criteria for the top ten nations. The methodology takes advantages of membership function for approximations. Therefore, objects with approximation can be assigned with a fuzzy number associating the decision.

3.1 Data preprocessing

The criteria of OECD well-being have the three challenges, i.e., inconsistency of influence directions, different scales, and missing data. These three can cause uncertainty, inconsistency, and ambiguity. The first originates from different influences such as negative and positive. For instance, the negative criteria q_5, q_6, q_8, q_{10}, q_{21}, q_{22}, and q_{23} generally pull down the well-being. The second challenge stems from normalization which might distort the scaling. The third faces missing values. In our preprocessing the first challenge is solved by reversing the preference orders. The third can be solved by representing approximation objects with the fuzzy numbers which are presented in Proposition 9. The top ten nations in 2012 are Canada, Denmark, Netherlands, Norway, Sweden, Switzerland, Australia, New Zealand, Luxembourg, and United States.

3.2 Assigning the fuzzy numbers for objects

Assignment of the fuzzy numbers for objects is designed from an information system composed of preferences. This research aims to transform the preferences into the fuzzy number and generate induction rules based on the numbers. Totally, there are 12 propositions for implementation. Propositions 1 and 2 are about the information system and preference expressions. Propositions 3 and 4 are about the induction rules, $q^{\geq}_{j,t'}\rightarrow Cl^{\geq}_t$ by DRSA, and the accurate coverage rate which presents the degree how much a criterion supports nations to achieve a dominating level [18]. Propositions 5, 6, 7, and 8 assign the fuzzy numbers to credibility of dominance unions, *P*-dominating set, *P*-lower approximation, and *P*-upper approximation. Proposition 9 presents a generalized assignment of a fuzzy number to an object.

Proposition 1: Information system of DRSA

$DRSA = (U, Q, f, V, Cl_t^{\geq})$ where $U = \{y_k \mid k = 1,...,n\}$, $Q = \{q_1, q_2,..., q_m\}$, $f : U \times Q \to V$, $V_Q = (V_{q1}, V_{q2},..., V_{qm})$, $Cl_t^{\geq}$ is a dominating union having nations at least not less than t, and t is a rank place like 10^{th}. This proposition transforms sets into an information system.

Proposition 2: Preference orders

$r_{xj} \succsim r_{zj} \Leftrightarrow f(x, q_j) \geq f(z, q_j)$, $\forall x, z \in U$ where f is a function that maps a criterion to a preference value for a nation. For instance, r_{xj} and r_{zj} are preference values of nation x and z with respect to q_j.

Proposition 3: A dominating rule

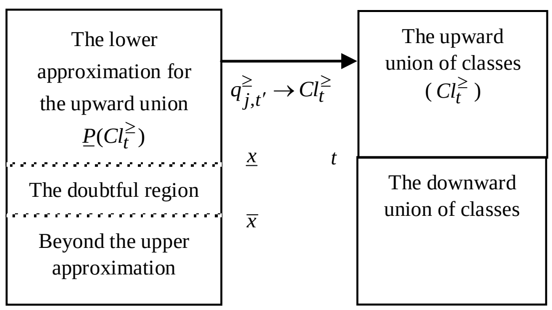


Fig. 3 A dominating rule for approximations

$q_{j,t'}^{\geq} \to Cl_t^{\geq}$ Represents how a criterion q_j support nations to achieve the top t positions where $q_{j,t}^{\geq}$ is a set of nations within the top t positions with respect to q_j. This rule associates a dominating set to a dominating union and is independent to addition or removal of other criteria. In our design the dominating rule can be conceptualized as Fig. 3. $\underline{P}(Cl_t^{\geq})$ is the lower approximation containing the boundary object $\underline{x}$ and objects at least as good as $\underline{x}$ in all considered criteria. The considered criteria belong to *P*. $\overline{P}(Cl_t^{\geq})$ is the upper approximation containing the boundary object $\overline{x}$ and objects at least as good as $\overline{x}$ in all considered criteria. $q_{j,t'}^{\geq}$ is an approximation, $(q_{j,t'}^{\geq} = \bigcup_{s \geq t'} q_{j,s})$, containing nations ranked in at least t' with respect to criterion q_j. The boundary objects $\underline{x}$ and $\overline{x}$ are presented as slash lines for $Cl_t^{\geq}$. To find out the positions for $\underline{x}$ and $\overline{x}$.

Proposition 4: An accurate coverage rate is defined as

$g_j' = g(q_{j,t'}^{\geq} \to Cl_t^{\geq})$ for $q_{j,t'}^{\geq} \to Cl_t^{\geq}$ which is a unique value to present the degree that q_j supports nations to compete the top t positions, $0 \leq g_j' \leq 1$. Its derivation is described in Model I.

Model I:

$$\text{Max } g'_j = CR(Cl_t^{\geq}) \times \alpha(Cl_t^{\geq})$$

$$\textbf{s.t. } \underline{P}(Cl_t^{\geq}) = D_P^{+}(\underline{x}),\ \overline{P}(Cl_t^{\geq}) = D_P^{+}(\overline{x}),\ P = \{q_j\}$$

$$CR(Cl_t^{\geq}) = \frac{|\underline{P}'(Cl_t^{\geq})|}{|Cl_t^{\geq}| - missing}\ ,\ \ \alpha(Cl_t^{\geq}) = \frac{|\underline{P}(Cl_t^{\geq})|}{|\overline{P}(Cl_t^{\geq})|}$$

where $CR(Cl_t^{\geq})$ represents a coverage rate that can handles data with missing values. The variable $missing$ is the number of nations within $Cl_t^{\geq}$ that has empty values with respect to the criteria in P. The missing here is processed by substituting a number 0 to make it invisible in the upward union. The size of the upward union thus can dynamically adjust and g'_j will change with the resizing of the upward union. One point to note is that rank of $x \geq$ rank of z with respect to P.

Proposition 5: The fuzzy numbers of dominance unions

$$Cl_t^{\geq}(x) = \begin{Bmatrix} 1 & x \in Cl_s,\ \textbf{rank of } s \geq \textbf{rank of } t \\ 0 & \textbf{otherwise} \end{Bmatrix}$$ which assigns a membership degree to the credibility for upward and downward unions. This value depends on user's knowledge and decision. All inductions of the fuzzy numbers depend on it.

Proposition 6: The fuzzy numbers of P-dominating set

$$D_P(y,x) = T_{q_j \in P}\{\succsim_{q_j}(y,x)\} = \begin{Bmatrix} 1, & y \succsim_{q_j} x \\ 0, & y \prec_{q_j} x \end{Bmatrix}$$

which assigns a membership degree to the credibility for dominating sets, i.e., 1 or 0. This assignment presents the outranking with the fuzzy numbers.

Proposition 7: The fuzzy numbers of P-lower approximation

$$\underline{P}[(Cl_t^{\geq}(x)] = \begin{Bmatrix} 1 & x \in D_P(x, \underline{x}) \\ 0 & \textbf{otherwise} \end{Bmatrix}$$

where $x,\ \underline{x} \in U$ and $\underline{x}$ defined in Fig. 3 represents the DRSA boundary for the lower approximation. It takes advantage of the lower approximation boundaries to give membership degree for objects. So, the objects within the lower approximation have the fuzzy number 1; otherwise 0.

Proposition 8: The fuzzy numbers of P-upper approximation

$$\overline{P}[(Cl_t^{\geq}(x)] = \begin{Bmatrix} 1 & x \in D_P(x, \overline{x}) \\ 0 & \textbf{otherwise} \end{Bmatrix}$$

where $\overline{x}$ defined in Fig. 3 represents the boundary of the upper approximation. There are two values for objects belonging to the upper approximation or not. The distribution of membership degrees is located within the dashed circle of Fig. 4.

3.3 A fuzzy number system based on DRSA

According to Proposition 7 and 8, each object belonging to the lower, the doubtful region or beyond the upper approximation can be assigned a fuzzy number associating the conditional criteria and decision.

Proposition 9: A generalized fuzzy number for an object is designed as:

$A_j(x) = 0.5 \times (\underline{P}[Cl_t^{\geq}(x)] + \overline{P}[Cl_t^{\geq}(x)])$ where $x \in U$, $P = \{q_j\}$, and $A_j = \{1, 0.5, 0\}$ where $j = 1.m$, $A_j = \{1, 0.5, 0\}$ where $j = 1..m$, $A_j = 1$ represents objects belonging to the lower approximation, $A_j = 0.5$ for objects belonging to doubtful region, and $A_j = 0$ for objects beyond the upper approximation. The generalized fuzzy numbers for objects thus can be built to process classification and reveal dominating characteristics.

3.4 Fuzzy number addition

The fuzzy number addition aims to give the aggregated and decomposed information for induction rules. It can give explanations for implications by the followings.

Proposition 10: Under the assumption of independent inductions, $q_{j,t'}^{\geq} \to Cl_t^{\geq}$, the fuzzy numbers crossing criteria can represent an accumulated membership degrees in wide perspectives. So, the higher accumulation the closer to the decision in wide perspectives an object can be. Therefore, an added fuzzy number crossing multiple criteria for an object's objective can be formulated as:

$$A(x) = \sum_{j=1}^{m} w_j A_j(x) \qquad (1)$$

where $A(x)$ functions as an accumulated membership degree for an objective, $w_j \in \{0,1\}$ and $x \in U$. w_j represents a reduction mark which can eliminate the criteria q_j by setting $w_j = 0$. The elimination depends on users' decision. For an instance of $w_j = \left\lfloor \frac{g'_j}{0.6} \right\rfloor$, $w_j = 0$ when $g'_j < 0.6$.

Proposition 11: Induction quality based on the added fuzzy numbers can be solved by model II which has been implemented successfully in Lingo 12.

Model II:

$Max\ g'_A$

$s.t.$

$g'_A = CR(Cl_t^{\geq}) \times \alpha(Cl_t^{\geq})$

$CR(Cl_t^{\geq}) = \frac{|\underline{P}(Cl_t^{\geq})|}{|Cl_t^{\geq}|}, \alpha(Cl_t^{\geq}) = \frac{|\underline{P}(Cl_t^{\geq})|}{|\overline{P}(Cl_t^{\geq})|}, P = \{A\}$

$A(x) = \sum_{j=1}^{m} w_j A_j(x), w_j = \left\lfloor \frac{g'_j}{0.6} \right\rfloor, w_j \in \{0,1\}, m = |Q|$

The value of 0.6 in $w_j = \left\lfloor \frac{g'_j}{0.6} \right\rfloor$ represents a fuzzy cut, i.e.,

${}^{\alpha}A_j = \{x \in U \mid A_j(x) \geq \alpha,\ \alpha = 0.6\}$.

Proposition 12: '*if* $A(x) \geq \beta$, *then* $x \in Cl_t^{\geq}$' is an induction rule based on the added fuzzy numbers. Users can also get the decomposed information for explaining the dominance characteristics like Table 3.

4 RESULTS

The resulted accurate coverage rates of 24 criteria of OECD well-being are lists in Table 2. The gray column is about the upper half level and white about the top ten. The higher accurate coverage rate means the corresponding criterion has stronger conditional dependence in well-being for nations. In statistics 12 criteria in the upper half level and 4 criteria in the top level have $g'_j \geq 0.60$. obviously the high well-being level has less accurate coverage rates. A nation intending to give satisfied well-being needs to pay attention to two stages. First, keep personal earning (q_4) stable and sustainable. Second, sustain employment rate (q_3), lower long-term unemployment rate (q_6), dwellings with basic facilities (q_9), and life satisfaction (q_{24}).

Table 2. The accurate coverage rates of OECD well-being 2012

Income			Community		
q_1	0.46	0.78	q_{17}	0.45	0.82
q_2	0.42	0.55			
Jobs			**Civic engagement**		
q_3	0.64	0.67	q_{18}	0.40	0.56
q_4	0.53	**0.94**	q_{19}	0.38	0.57
q_5	0.33	0.50			
q_6	0.68	0.57			
Housing			**Environment**		
q_7	0.46	0.68	q_{20}	0.50	0.78
q_8	0.31	0.54	q_{21}	0.35	0.57
q_9	**0.77**	0.63			
Work-life balance			**Safety**		
q_{10}	0.33	0.55	q_{22}	0.31	0.57
q_{11}	0.40	0.52	q_{23}	0.34	0.55
Health			**Life Satisfaction**		
q_{12}	0.38		q_{24}	0.68	0.82
q_{13}	0.59	0.76			
Education					
q_{14}	0.40	0.64			
q_{15}	0.34	0.55			
q_{16}	0.37	0.65			

Followings present the induction rules by Proposition 12.

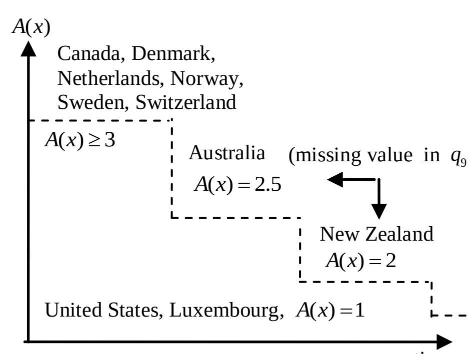
Rule 1: *if* $A(x) \geq 2$, *then* $x \in Cl_t^{\geq}$ where $g'_A = 0.71$, $Cl_t^{\geq}$ include the top ten nations,

$P = \{q_3, q_6, q_9, q_{24}\}$, $\underline{x} = \{\text{New Zealand}\}$,

$A(x) = A_3(x) + A_6(x) + A_9(x) + A_{24}(x)$ In Rule 1, 80% nations in the upward union are covered, i.e., Canada, Denmark, Netherlands, Norway, Sweden, Switzerland, Australia, and New Zealand in white background of Table 3, located above New Zealand in Fig. 4. New Zealand places at the fuzzy cut position with $A(x) = 2$. Two outliers are Luxembourg and US.

Table 3 Fuzzy numbers for Rule 1

	q_3	q_6	q_9	q_{24}	$A(x)$
Netherlands	1	1	1	1	4



Norway	1	1	1	1	4
Denmark	1	0.5	1	1	3.5
Canada	0.5	1	0.5	1	3
Sweden	1	0.5	1	0.5	3
Switzerland	1	0	1	1	3
Australia	0.5	1		1	2.5
New Zealand	0.5	1		0.5	2
Luxembourg	0	1	0	0	1
United States	0	0	1	0	1
Count of $A(x)=1$	5	5	5	6	
Count of missing	0	0	2	0	
Count of $A(x)=0.5$	3	2	1	2	

Rule 2 reveals that United States and Luxembourg perform well in households' income (q_1) and voter turn-out (q_{19}). Its formula is presented as:

Rule 2: *if* $A(x)=2$, *then* $x \in Cl_t^{\geq}$ where $g'_A=1$, $A(x)=A_1(x)+A_{19}(x)$, and $Cl_t^{\geq}$ ={Luxembourg, United States}

Rule 1 and 2 represent two concepts of well-being. Rule 1 comprises eight nations focusing on jobs, housing, and satisfaction. It illustrates the major characteristics by the aggregated numbers. Rule 2 focuses on income and civic engagement which covers a minor part of the top ten nations. The application results reveal decomposed information and the aggregated characteristics in of the top ten well-being level. The merits of FNA have two points. First, transforming approximations into the fuzzy numbers makes classification calculated by fuzzy operations. Second, the fuzzy numbers can reveal the aggregated and decomposed information for dominating characteristics which is discussed next section.

5 DISCUSSION AND IMPLECATION

The operations of the fuzzy numbers instead of preferences are much easier. The advantages are lists below.

- Accumulated well-being is available. For instance as Proposition 12, $A(x)=\sum_{j=1}^{m} w_j A_j(x)$ where $A_j(x)$ assumes a membership degree for nation x with respect to criterion q_j and $A(x)$ represent the accumulation for nation x.
- Preferences of accumulated well-being are available for induction. According to Proposition 12, the outranking relationship can be expressed as

$$u(x) \geq u(y) \leftrightarrow \sum_{j=1}^{m} w_j A_j(x) \geq \sum_{j=1}^{m} w_j A_j(y) \qquad (2)$$

- Missing data can be handled dynamically in inductions. For instance, Australia belongs to the top ten nations while her dwellings with basic facilities (q_9) is missing. By FNA, she is still placed at the top positions due to her high sustainability of the employment rate (q_3), long-term unemployment rate (q_6), and life satisfaction (q_{24}). Therefore, a prediction that Australia has a good performance in dwellings with basic facilities (q_9) is very possible. Alternatively, New Zealand has her life satisfaction (q_{24}) a little bit lower and dwellings with basic facilities (q_9) missing. Her prediction in dwellings with basic facilities (q_9) is not as sure as Australia.

- Well-being classification becomes easy by taking advantages of criteria elimination and the fuzzy numbers addition. For instance, FNA can process criteria reduction in an easy way, i.e., eliminating criteria having too few of $A_j(x)=1$ or too many of $A_j(x)=0.5$. Technically, criteria with a small size of the lower approximation or a big size of the doubtful region can be eliminated by checking small g_j. There are 20 criteria eliminated here. An elimination summary is presented in Table 4. Furthermore, the fuzzy numbers addition makes induction as easy as a fuzzy cut on the added number. Users just need to sort Table 3 then find out a cut for classification.

Table 4 Eliminated criteria for the top ten level

	Eliminated criteria
Small size of the lower approximation ($\lvert A_j(x)=1 \rvert \le 5$)	q_4, q_5, q_7, q_8, q_{11}, q_{14}, q_{18}, q_{20}, q_{22}
Big size of the doubtful region ($\lvert A_j(x)=0.5 \rvert \ge 6$)	q_1, q_2, q_{10}, q_{12}, q_{13}, q_{15}, q_{16}, q_{17}, q_{19}, q_{21}, q_{23}

- The decomposed information from the FNA results can signify criteria with column summation. For instance, by excluding the outliers the count of $A_j=1$ for q_3, q_6, q_9, q_{24} are 5, 5, 5, and 6. Obviously, the most significant criterion is life satisfaction (q_{24}) which has six nation with $A_j=1$ in the lower approximation. The second significant is long-term unemployment rate (q_6) which has fewer $A_j=0.5$ than q_3 in the lower approximation. Dwellings with basic facilities (q_9) is the least important due to more missing data.

The comparison among DRSA, the fuzzy integral, and FNA is presented Table 5. These three methods have the same inputs, preferences and dominating union. In the comparison of the outputs, their difference centers at the capability of the aggregated and decomposed information. DRSA can provide decomposed information while no aggregated information. The fuzzy integral has reverse performance which has the aggregated information but no decomposed information. FNA combines both merits thus has an advantage over the other methods.

Table 5 Classification comparison (multi-criteria)

Input requirements	DRSA	FI	FNA
preference scales	Y	Y	Y
Dominating union	**Y**	**Y**	**Y**
Output performance			
Classification	Y	Y	**Y**
Aggregated inf.	N	Y	**Y**
Decomposed inf.	Y	N	**Y**

The case study about the top ten well-being level has three points.

- Satisfying people with living demands and happiness feeling is a key to move nations to a better well-being. For instance, employment rate (q_3) and long-term unemployment rate (q_6) belong to living demands. Dwellings with basic facilities (q_9) and life satisfaction (q_{24}) belong to happiness feeling. According to Table 3, Canada and Australia are not so good in the employment rates but their low long-tern unemployment rates make their people satisfied with life. Conversely, Denmark is not so good in the long-term unemployment but her high employment rate makes her people satisfied with their living. The long-term unemployment (q_6) functions a complement role to the employment (q_3).
- It does not guarantee people as happy as their income For instance, United States and Luxembourg in the gray background of Table 3 are ranked in the top ten positions by OECD. They did not achieve the fuzzy cut, $A(x) \geq 2$, in Table 3 but performed well in households' income (q_1) and voter turn-out (q_{19}).United States and Luxembourg have high income and better life however their people are not as happy as their income. It seems that the personal income has stronger affects than the unsatisfied economic situation which deserves further exploration in the future.
- Paying attention about job creation and interview for weak forces makes people feel well-being. For instance, Denmark and Switzerland tried to be good in job creation and interview for weak forces even their long-tern unemployment is high. Their people still feel satisfied with their living.

6 CONCLUDING REMARKS

This research proposes FNA for OECD well-being in 2012. There are four merits achieved in this empirical research. First, objects within approximations are assigned with the fuzzy numbers which make a homogeneous-scaling for an information system. The fuzzy numbers are built by associating the conditional and decisional criteria. The complexity of preference function and the corresponding values for objects are reduced. Second, the fuzzy numbers addition combined with fuzzy cuts successfully generates induction rules. Therefore, the approximations can be explained by the aggregated and decomposed information of approximation. Third, the case study shows 80% of the top ten nations in the top ten level of well-being are covered by FNA rules. They are Australia, Canada, Denmark, Netherlands, New Zealand, Norway, Sweden, and Switzerland. These eight nations performed well in employment rate (q_3), long-term unemployment rate (q_6), dwellings with basic facilities (q_9), and life satisfaction (q_{24}). The other two nations, United States and Luxembourg have good points in households' income (q_1) and voter turn-out (q_{19}) however people were not so happy as their income. Forth, a well-being strategy for a nation can follow the resulted inductions, i.e., keep well in personal earning (q_4) then sustain employment rate (q_3), long-term unemployment rate (q_6), dwellings with basic facilities (q_9), and life satisfaction (q_{24}).

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